Industrial and Systems Engineering Seminar

Efficient Simulation of Risk and its Error: Confidence Intervals for Quantiles When Using Variance-Reduction Techniques

Wednesday, March 13

3:15 PM – Refreshments before the Seminar

3:30 PM – Graduate Seminar

Mechanical Engineering Room 4125 A & B

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The p-quantile of a random variable is the constant for which exactly p of the mass of its distribution lies to the left of the quantile; e.g., the median is the 0.5-quantile. Quantiles are widely used to assess risk. For example, a project manager may want to determine a time T such that the project has a 95% chance of completing by T, which is the 0.95-quantile. In finance, where a quantile is known as a value-at-risk, analysts frequently measure risk with the 0.99-quantile of a portfolio's loss; thus, there is a 1% chance that the loss will be greater than this value. For complex stochastic models, analytically computing a quantile usually is not possible, so simulation is employed. In addition to providing a point estimate for a quantile, we also want to measure the simulation estimate's error, and this is typically done by giving a confidence interval (CI) for the quantile. Indeed, the U.S. Nuclear Regulatory Commission requires that licensees of nuclear power plants demonstrate compliance using a "95/95 criterion," which entails ensuring (with 95% confidence) that a 0.95-quantile lies below a mandated limit.

In this talk we present some methods for constructing CIs for a quantile estimated via simulation. Unfortunately, crude Monte Carlo often produces wide CIs, so we incorporate variance-reduction techniques (VRTs) in our simulations to decrease the error. We first discuss forming a CI using a finite difference, and the second approach applies a procedure known as sectioning, which is closely related to batching. The asymptotic validity of the CIs follows from a so-called Bahadur representation, which we have established for a broad class of VRTs, including antithetic variates, control variates, replicated Latin hypercube sampling, and importance sampling. We present some empirical results comparing the different CIs.

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Marvin K. Nakayama is a professor of computer science at the New Jersey Institute of Technology. He received a B.A. in mathematics-computer science from U.C. San Diego, and an M.S. and Ph.D. in operations research from Stanford University. He won second prize in the George E. Nicholson Student Paper Competition sponsored by INFORMS and received a CAREER Award from the National Science Foundation. He is the Simulation Analysis and Stochastic Modeling Area Editor for the *ACM Transactions on Modeling and Computer Simulation*, and the Simulation Area Editor for the *INFORMS Journal on Computing*. His research interests include simulation, applied probability, statistics, risk analysis, reliability, and modeling of cascading failures.